# ON GRAVITATION AND RELATIVITY

BEING

THE HALLEY LECTURE

DELIVERED ON JUNE 12, 1920

BY

RALPH ALLEN SAMPSON, D.Sc., F.R.S.
*Astronomer Royal for Scotland*

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## ON
# GRAVITATION AND RELATIVITY

THE idea of gravity presented little difficulty to the ancients. Each existence was ruled by its peculiar law. Even lifeless things were animated by a power—whether it originated within them or without them—which impelled them to seek the centre of the world. Thought appeared here to run in an easy familiar groove, but the theory of a moving earth upset it. For, why did things seek to reach its centre if the Earth was not the centre of the world, but merely a minor satellite of the Sun? The difficulty called for a new revolution of ideas, it echoed afresh Copernicus's quotation

*provehimur portu, terraeque urbesque recedunt.*

We are swept from our anchorage indeed, the lands we knew, the cities we dwelt in, fade into mist.

Metaphysical juggling with physical matters was at that time a staple of the universities, and Galileo was a master juggler. He gives its proper prominence to the feature I refer to in his *Dialogues* on the systems of Ptolemy and Copernicus. He uses all his art to expose the illusions and the tricks current among rival professors of his craft. We see the doomed theory as a tottering, impossible, ridiculous edifice. To the seed spread by this book, fostered by the immense notoriety of its author's prosecution, one is perhaps not wrong in tracing the growth, essentially British as it is, of the study of the positive in Natural Science. Oxford took

an active part in it: the characteristic motto of the Royal Society well expresses it, *Nullius in Verba*, which I translate 'Wranglers, avaunt!' It was made good in their early Proceedings, where the hoary, accumulated traditions and travellers' tales of past centuries were put to experimental proof and swept away.

We know that Newton as a student eagerly read these Proceedings. There can hardly be a doubt that he read as eagerly Galileo's *Dialogues*, then widely current in many editions and in Salusbury's classical translation. No need for an apple to fall upon his head to set his thoughts in motion if he found the following:

> Salviatus. . . . I say that that which maketh the Earth to move, is a vertue, like to that by which *Mars* and *Jupiter* are moved, and wherewith [he] believes that the starry sphere it self also doth move; and if he will but assure me who is the mover of one of these moveables, I will undertake to be able to tell him, who maketh the Earth to move. Nay more; I will undertake to do the same, if he can but tell me who moveth the parts of the Earth downwards.
>
> Simplicius. The cause of this is most manifest, and every one knows that it is gravity.
>
> Salviatus. You are out, *Simplicius*, you should say, that every one knowes, that it is called Gravity: but I do not question you about the name, but the essence of the thing, of which essence you know not a tittle more than you know the essence of the mover of the stars in gyration; unless it be the name that hath been put to this and made familiar and domestical, by the many experiences which we see thereof every hour in the day, but not as if we really understand any more, what principle or vertue that is which moveth a stone downwards, than we know who moveth it upwards when it is separated from the projicient, or who moveth the Moon round, except—as I have said—only the name, which more particularly and properly we have assigned to the motion of descent, namely, Gravity. . . .

Once more a period of change and inquiry has come, and we are told that the lands and cities that Copernicus and Newton have led us to are not permanent dwelling-places. Once more we are bidden to cast loose from our anchorage in space, and adventure on an uncharted ocean to discover new and strange continents. Once more we must endeavour to restate the metaphysical postulates of argument as Newton so profoundly and adequately stated his own. And once more astronomy is the arbiter and guide.

Newton always professed a distaste for speculation, as ending in idle wrangling, and within the covers of the *Principia* he eschewed it altogether. Yet his speculations upon subjects which to us also are still matters of speculation, are among his most interesting legacies. There is one of these, in a letter to Robert Boyle,* sketching the cause of gravitation as he pictured it to himself, which I would make the support for the few uncompleted remarks I am going to offer—remarks which claim no merit beyond that of presenting difficulties where I see them.

Newton conceived gravitation to be due to pressure by the aether. His aether was atomic, and in a normal state exerted pressure upon matter, somewhat after the manner of gas-pressure, but working, not as we now conceive gas-pressure to work, by impacts of its moving molecules, but by some form of elastic resilience at rest. These aether atoms penetrate the pores of matter, and as they approach any material condensation they become rarer in distribution, and their resilience is less. Thus any two bodies find themselves under pressure upon all sides, but always less on the sides they present

* *Macclesfield Correspondence*, vol. ii, p. 407. Printed also by Horsley and Brewster.

to one another. They are therefore impelled towards one another, and this is the phenomenon of gravitation. Newton did not assign any law to the rarefaction of the aether, which occurs in the neighbourhood of matter, but this is easily supplied. If we suppose mass in matter is merely representative of the numerical collection of equal ultimate particles, and if for each particle the reduction of pressure induced in the surrounding aether is inversely proportional to the distance from that particle, that is to say, if $p = constant - m/r$, then Newton's conception of an aether-pressure is in every respect identical with his law of gravitation.

Let us then conceive of atoms of matter as singularities movable in the aether, and, wherever they go, relaxing its spring or potential energy in their own neighbourhood. Let us employ this conception in a few rough illustrations, in order to mark the character of the phenomena that fall to be explained and the number of indispensable elements that must be introduced for the purpose.

Let us economize by treating of space of two dimensions. We then have a third dimension at disposal, which may be used to indicate relative potential energies, measuring the essential tendencies that bodies have, to move to other positions. For let it be admitted, as it has been proved by many an ancient paradox, that events conceived in terms of space alone leave motion incomprehensible. Motion is an independent essential element in phenomena. Nature's garment that we know her by is a moving, living thing—*der Gottheit lebendiges Kleid.* Motion, or the tendency to motion, or any other manifestation of the time element, is the weft of this garment and space is its warp; the fabric does not exist if one of them is absent.

Imagine, then, the aether as an elastic skin, stretched, let us say, on the surface of a sheet of water, the depth of the water below it serving as an index of potential energy. An atom of matter, which is characterized by the property of relaxing the potential energy in the surrounding aether, would figure in this model as a heavy particle resting on the skin and producing a small pocket or depression in the water surface.

If we have two material bodies, the aether pockets, associated with each, would superpose their depressions, with the result that the bodies would tend to move towards one another. Each tends to fall into the pit made by the other. This is gravitation. If they possessed any relative motion transverse to the line between them, they would circulate round one another. It is an easy matter to write down the equations of motion of a single minute particle on the surface of such an aether pocket; they differ from the familiar forms for a plane orbit in introducing what may be considered as a double measure of distance, namely, distance from the axis of the pocket, and distance measured along its curved surface. In consequence of this feature, if the grading of the potential energy in the pocket is adjusted so as to ive, say, a law of attraction according to the inverse square for motion in a direct line towards the centre, when there is transverse motion a closed orbit will *not* be described. The pure law of inverse square is the only law we need consider, valid for all distances, that will keep the apse fixed. Change it in any small respect and the apse will move. This is the phenomenon known as the progression of the apse.

Now take another feature—the transmission of light. This must be imagined as a wave transmitted in and by the aether skin. The velocity, which will be greater or

less according as the local resilience in the aether is greater or less, will always be less when the wave passes the relaxed region that is near matter. In consequence of this the wave-front will wheel round matter that it passes like a file of soldiers turning a corner, those near the pivot moving slower. This is the gravitational deflection of light.

We are not yet done with our model. The use of aether pockets and a three dimensional space figure to explain gravitation in a two dimensional space, might be described as explaining it in terms of a curvature of space. The double measure of distance referred to above throws light upon this description. It is nothing but a form of words. No curvature of the two dimensional aether-field can be ascertained unless we retain a flat two dimensional space-field to compare it with. The double measure of distance must be retained, but it may all be on the flat, the one measure being a geometrical measure independent in scale of the place where it is applied, and the other based, say, upon counting the aether atoms or some other suitable mechanical feature around the particle, these being differently spaced as we recede from its neighbourhood.

An experiment on these lines is easy to perform with balls of paraffin wax floating in water, and supported by the surface film that they try to drag down with them. The pits that they form in the water will seek one another out over considerable distances. The theory of the phenomenon in this actual case would, of course, require a slightly different wording, but merely to the extent of referring to the potential energy of the film as well as that of the pit in the water.

Needless to say, I do not bring this experiment before you as something new. I bring it as something old.

I bring it too as illustrating the amount of tautology which is present in almost all we can say in explanation of gravitation. An explanation is the exchange of two statements for one. To exchange one statement for another is no explanation. It may be considered a simpler view to accept an energy relation between *n* separate bodies and one universal medium than to regard the more numerous separate relations between these bodies without such a medium, but there is no more in it than that. The ultimate acceptance of the possession by each body of some virtue, however described, which induces other bodies to move towards it, is present here, and it is none the less present in any other theory that has been offered.

It would be idle to cross-examine our rough hypotheses and ask if they can supply results metrically correct. If they did, it would be no confirmation of them. Needless to say, they are not in the least intended to compete with the theory in which Einstein has put all he has to say, without redundancy, defect, or effort, into one consummate formula. Their poverty and clumsiness, their age and obviousness, are intended to show how much of the intelligible outcome may be found in the crudest theory we can enunciate, that attributes gravitation to the medium.

There never was a bolder theory than that with which Einstein met the difficulty left by Michelson and Morley. If it were not for his adventures—raids—or conquests in the territories of Space and Time, the subject would hardly have been heard of in the greater world. As it is, because of its boldness, its range, its brilliance and resounding successes, all who are interested in ideas must endeavour to grasp its position.

It is algebra,—that is the key. The force as well

as the beauty of algebra comes from its symmetry. By its aid, the magician with his wand can order hosts as easily as if they were units. Small blame, then, to mathematicians who have a passion to symmetricalize their data. But they do it at their own risk. The portrait of Nature that they supply, only interests the investigator of Nature in so far as he can recognize the lineaments. The mathematician's word is not enough, when he bids us to merge time and space into one medium, to bend our minds until we can conceive a curvature in them both, and to picture each as a complex quantity possessing imaginary as well as real parts. The plan must be examined on its merits, for we know that he has adopted it merely for its convenience. It is already a great demand, to admit positive and negative in Time and so to obliterate in principle the distinction between past and future. It is parallel to the old-fashioned convention of electrostatics now known to be so misleading, which regarded positive and negative in electricity as a mere mathematical antithesis. The prominence of such instances as the swing of a pendulum, where the reversibility is pretty nearly perfect, is purely due to selection. Are we to admit the possibility, as Lord Kelvin put it, that a man could live backwards and become again unborn? For all that Nature bears upon her face, phenomena may only allow of being linked in chains of causation, one way round.

We should, of course, make next to no progress, if we insisted on taking no step until we were ready to prove it was right. When Newton affirmed the law of gravitation—verified as between Sun and planets—for every particular atom of matter, it must be admitted his grounds for doing so were far from demonstrative. He said it was rendered general by induction. He was

carried forward by an instinct for mathematical symmetry. In the same way, it must be admitted that Einstein has speculated, as genius only is justified in doing, and only genius by success. Thus take first what may be described as the motive power in his mathematical machinery—the Principle of Equivalence, which allows him to search his field with the processes and results of the theory of invariance. If it be surmised that the difficulty we are examining has its source in confused communication of elementary notions and measures of space and time as between A and B, we may well suppose that it would disappear if we could assign the equation, or substitution, that expresses A's estimates in terms of B's. But may we then affirm that for a fundamental fact of Nature, like gravitation, only that part and version of our expression is real which survives in invariant form a general algebraic substitution—that substitution moreover embracing time and space undistinguished in the same category? The mathematician will take that course without fear, for he knows that his tall ship can ride the deep seas; he knows he will wreck her at once if he follows the variety of Nature's coastline, he is safe when he is out of his depth, but who will be inclined to follow him? Nature does not possess the generality of an algebra. On the contrary all its manifestations are extraordinarily special. That is their peculiar charm. Birds, insects, animals, the rocks, the trees, appear to be mere fragments and accidents, compared to the variety that might have been. And if the particles of which they are composed approach closer to elementary uniformity, and the notions of space and time which lie behind them are more uniform again, still one remains at a loss to conceive any region that may correspond to the imaginary field of the algebraist.

Nonsense of the purest water can be derived from treating space and time on a united footing. We know how to rotate a body about a point or an axis. In four dimensions we must learn to rotate it about a face. The mind does not respond to the suggestion. No matter. Let us suppose it done. We turn a cube over, and it becomes a square multiplied into a period of time. If that appears obscure, you can illuminate it by recalling that the period of time belongs to an inhabitant of Mars. A quart consumed to-day may perhaps be proved congruent to a pint consumed to-morrow. The Turtles' Academy in Looking Glass Land is the place where these propositions belong. It is a thousand pities that Lewis Carroll did not live to exploit their possibilities.

But this is not the only paradox which Einstein's theory carries, and, be it not forgotten, carries to success. The foregoing relates to its process. Its physical basis is the deduction, or generalization, or speculation, from the Michelson-Morley experiment—that each separate observer possesses his own peculiar notions and standards of space and time, which are incapable of telling him whether he is himself in motion, and which are not communicable to another so as to permit adjustment, except indeed as so far as the velocity of light appears to each under all circumstances the same.

To have made out of this material a theory which is certainly coherent and will stand examination is, to say the least, an astonishing mathematical feat. It would seem, one would say, to float in the void, like a meteor in space. It is true that it too may be forced into ludicrous antinomies with which some of its disciples, possibly unused to metaphysics, and suddenly speaking with tongues, have perhaps more surprised than edified their hearers. But the theory is possible, though it is

difficult, and in some cases its steps have been actually performed. And, as we all know, it has been shown to contain, not merely descriptively, but exactly and metrically, the rate of progression of the perihelion of Mercury and the gravitational deflection of light.

Now can we gather grapes from thorns or figs from thistles? That is the question; or rather the question is—Is this particular fruit a fig, or is it an apple of Sodom? Before expressing an opinion I would ask you to regard from a different standpoint the question of determining absolute position, and that of communication between observers.

To say that two observers, $A$ and $B$, separated from one another, can have no common verifiable standard of time, is to contemplate a very special set of circumstances. It is indeed true if they are receding from one another in a straight line, for their comparisons are complicated by the time of transmission of the signals, and cannot be cleared from it unless the circumstances can be repeated with some variation. But this case is merely a mathematician's abstraction. He has, moreover, abstracted from it everything that makes it an actual case. Our knowledge of Nature is not reached by clean-cut steps of ratiocination; least of all our primary perceptions. Only by continual repetition are permanent ideas detached, at last, and imperfectly, from an intricate, unremembered background of the mind. We should know nothing at all about such circumstances. There is no knowledge without repetition, or without memory, or the time element in some form. No repetition is possible when two bodies are simply moving apart. Now we are supplied with many natural measures of time, and many of these are capable of common use. We have the pulses of our body, which we can use, as

indeed Galileo used them when he timed the swing for different arcs of the chandelier in Pisa Cathedral. Some people have the gift of absolute pitch in music, which is a time measure, and which they constantly employ in harmony with others. One even might add temperature, the constant temperature of the body, which physicists recognize as a velocity datum. But the commonest of all is the rotation of the Earth, subdivided by the beats of a pendulum. Consider then the bearing of a rotating system, or of any repeating system, on the question of absolute, or at least consistent, determinations of time and space.

In a matter like this, where the metaphysical basis of theory is in dispute, decidedly we should begin with experiment. There are two experiments, simple, old, classical, familiar, which should be kept constantly in mind. The first is Newton's experiment of the rotating bucket, to demonstrate that absolute rotation can be perceived, though relative rotation may be insensible, quite contrary to what might be guessed from the analogy of linear motion. He tells us he performed it repeatedly. Evidently he attached considerable importance to it. Fill the bucket with water, set it in rotation and allow it to communicate its motion to its contents. At first the water appears undisturbed relative to the bucket though there is actually relative rotation; later, when bucket and water move together and there is no relative rotation, the absolute rotation of the water may be perceived by the water hollowing in the centre and creeping up the sides of the bucket.

The second is Foucault's pendulum. If the Earth were completely covered with cloud, and neither Sun nor stars were ever visible, its axis of rotation, and its rate of rotation—relative, it would seem, to an absolutely

external universe—could be correctly determined, within the limits of the experiment, by swinging a freely suspended pendulum. No conceivable gloss can deprive these experiments of their plain significance, and it is this: The data of Nature which are within our reach are such, as regards the motional element, that we can perceive the axis, and the rate, of a rotating body, with regard to fields that may be otherwise entirely outside of our ken. This is a piece of knowledge at least as striking as any of the negations of relativity. Admitting it—and who is there that questions it?—we may be permitted to proceed with the argument more succinctly with a construction that is indeed imaginary, but is so simple that I doubt if the most thoroughgoing Relativist will question what would be its outcome.

Imagine two concentric spheres, of radii virtually equal. $A$ is an observer on the outer surface of the inner sphere and $B$ is an observer on the inner surface of the outer sphere. The surfaces of the spheres that are presented to one another are featureless and perfectly reflecting. Let $B$ in the first instance be diametrically opposite to $A$, and let $A$ send out a light signal. The waves will be reflected at the outer sphere and, I say, will converge together at the same phase on $B$; they will then diverge again and will converge a second time on the point from which they were emitted, again together and in the same phase. But if the inner sphere is in rotation and $A$ has moved from $A_0$ to $A_1$ in the interval, it is evident, unless this result can be denied, that the observer $A$ can identify the point, absolute with respect to the medium transmitting the signal waves, which he occupied at a specific earlier moment.

There is nothing in this of a surprising character or inconsistent with other experiences of rotation, but it is

much at variance with the implications of Relativity. Let me then add two further points. The first relates to the doctrine of the essential relativity of time, both as to its zero and as to its scale. The scale is much the more interesting and important point. In order to secure the adopted canon of relativity—that two observers *A* and *B* shall always attribute the same value to the velocity of light, even when one of them is himself moving through the aether with that velocity—the intrinsic scale of time measurement is itself made dependent on the motion. This is the subtlest and boldest of all Einstein's ideas. A curious result follows. *A* and *B* both measure the time interval between two events and report to one another. *A* considers *B's* scale of measure is too small and *B* considers *A's* scale is too small. Evidently we must be prepared for paradoxes. The nearest analogy we can find is in geometrical perspective. *A* and *B* are separated by a few yards, and each holds out in front of him a yard measure. Perspectively *B's* measure appears to *A* smaller than his own, and *A's* appears to *B* smaller than his own. The solution is not remote. They have not used satisfactory means of communication. By stepping up to one another and superposing the scales, this individual geometry of the solipsist in which each has been indulging may be exchanged for one common to both, with a great enlightenment of view besides, when they reflect how the converging representation summed up on the tiny surface of the retina, has hitherto represented for each, differently, the infinite expanse of external space with its three dimensions. Now let our two observers be, *A* at Greenwich and *B* on the Moon. *A* observes the transit of *B*, and *B* observes *A* on the terrestrial meridian bisecting the Earth's disk. If there

is such a thing as a geometrical event, these are simultaneous. Each observer would time the occurrence, not right but a little late, by the same amount. By taking this observation repeatedly each would have the same time-interval, a mean lunar day, for common use. It is only if they confined themselves to signals which did not admit of the repetition that eliminates irrelevant admixtures that they would suffer under a 'relativity' in their time measures. That is the first point I would make.

The second is a question: How can our experience from rotation so falsify the notions of relativity derived from translation? In the example of the pair of spheres, it is plain that there is a proportion of arbitrary construction. There is no need to consider the motion of the inner sphere either as uniform or as taking place about a fixed axis. Let $A_0$ $A_1$ $A_2$ $A_3$ be any four points on the path of $A$; describe a sphere through them and complete the construction as before, but make these the points occupied by $A$ at successive returns of the signal after girdling the globe. In this way any absolute path of $A$ can be traced in space step by step, *except in the case when any three of the four points lie in a straight line*, that is to say, except for linear motion. For linear motion, then, the construction is impossible; the repetitions upon which we are dependent do not take place.

For linear motion, then, it is reasonable to expect that the degree of determinateness should be less, and that we should be unable to eliminate from our conclusions certain irrelevant features. If such a feature is unexpected, it may declare itself in the form of a paradox. Such cases are well known in mathematics, where the determinateness of a solution breaks down owing to some relation among the data. They are known by the

name of Porisms. Euclid studied them. Einstein's theory of relativity expresses the conclusions that follow from supposing that linear motion is our only standard. It is a porism—the most gigantic porism the world has ever seen or is likely to see.

Porisms have probably rightly been regarded as the severest test of a mathematician's power and dexterity. But they lose some of the interest attached to solutions which are at the same time complete and indeterminate, if we are able to make a supplementary statement which causes their indeterminateness to disappear.

It must not be supposed that in confronting the new theory with the facts of rotation, we are putting an unfair strain upon it. True, it is derived from considerations of linear motion alone, but the immense interest of relativity is that it disturbs the very basis of knowledge and asserts that we *can* know nothing of position in space and time; nor of space and time themselves except as merged into one medium. It might not be difficult to admit that there are regions of knowledge to which those statements apply, as in fact various familiar formulae of dynamics have been deduced from Einstein's equations, but it is as nearly obvious as anything can be, that rotation is a phenomenon that does not possess this featureless symmetry. Rotation proves the existence of a domain in which the law of relativity does not run. Now, periodic motions of all kinds, vibrations, oscillations, waves, the rotation of the Earth, the celestial motions, are the best known of all motional phenomena, for the reason that they offer themselves for repeated examination and permit the removal of irrelevant features. We cannot leave them out if we would. However coherent the theory of relativity may be in the regions which it claims as its

own, it is intolerable to make everything subject to it. Rather I submit that its two great postulates must be taken as defining and limiting the region of its validity, and further, that the expanse of this region must be found by trial and is certainly not co-extensive with the whole.

But take care lest we say too much. If the theory of relativity is a gratuitous mathematical feat, what becomes of the formidable Michelson-Morley experiment which it undoubtedly explained? What is the meaning of Einstein's two brilliant successes in astronomy? Once the ship is afloat what does it matter how it was launched? Or, to go back to the figure with which we started, if we conclude that we need not one and all embark upon the ship, and leave the cities and the lands we dwell in, shall we not still find that ship very useful for plying to and fro? We may not be forced to abandon our notions of space and time, such as they are, and adopt others which, be it confessed, are at present far from clear, but we still have those obstinate physical facts to explain, and can we explain them any better?

I will put you a conundrum: If a theory explains a fact, what is the position of the theory? It would be a simple mind that concluded that the theory was true. If the theory is incomplete in premises or definition, almost no binding conclusion can be drawn, except by cumulation of cases; and this is the position in regard to the vast mass of our knowledge of Nature. A theory such as that of Evolution commands for a period universal assent, but later its unexplored contingencies seem more and more unmanageable, and after the better part of a century of study it is found that though the theory is held more firmly than ever *in some form*, its actual

process remains in darkness. But the position is somewhat different with an exact theory, such as Einstein's relativity. Relativity is not a complete theory but it is an exact one. I have not myself succeeded in forming from it any idea of the nature of light; there seems no room in it for wave features, or interference, or polarization. It has less the character of a Physical Optics than of a Geometrical Optics. But if it is an outline merely, it is a firm outline, and its contingencies are presented all at once. Apply such a theory in explanation of a fact, and we *can* draw a definite conclusion—namely, that the fact does not as a necessity imply any more comprehensive postulates than the theory presents. When the fact is a very general one, such as gravitation, and the postulates are no more than that attenuated medium in which Einstein has succeeded in floating his theory, this conclusion is more weighty than might be expected. I began by showing how the descriptive facts of gravitation might be inferred from a crude construction of aether pockets associated with material nuclei, and further how we cannot keep from talking perfect nonsense if we meddle with the distinct presentation of time and space. Einstein has proved the irrelevance of this and all kindred theories and cautions by showing that, given the fundamental constant, all known features of gravitation may be foreseen exactly and calculated numerically from a coherent formula which asks for no mechanical model or diagram, no datum line in time or space, and no distinction in kind between them.

His theory breaks through the 'dome of many coloured glass' with which Time 'stains the white radiance of eternity'. But, we do not arrive at a perfectly white radiance. Something must remain to give us numerical results. Something akin to aether pockets of specific

depth figures in the theory, characterizing the presence of matter. Einstein describes these as a curvature of the space-time medium, but I regard this description as otiose. It is a return by another road to the concrete, from which his mathematics has just liberated him. It merely illustrates the indelible character of the mind's preoccupation with the spatial presentation of phenomena. If it is the aim to say the last word on gravitation, surely that goal is reached when it is shown to be contained in a formula, the elements of which do not of necessity bear *any* interpretable meaning. *Nec fas est propius mortali attingere divos.* We have reached Kant's *Ding an sich.**

The matter may then be fairly said to lie outside our ken. We have been used to speaking of gravitation as an ultimate property of matter. Einstein's work appears to me to show that it may be involved in the recesses of the nature of things in depths where our senses can never follow it. *May be,* not *Is.* Other experiences may induce us to accept a definite picture of the constitution of matter, and any scheme is possible that can be superposed to the phantom to which Einstein's profound dialectic has now reduced gravitation.

But may I add, to recover our breath, I am not convinced that the day of usefulness is over for those unpretentious illustrations which take up a subject in the middle and leave off as soon as they are beaten? They are like pacemakers in a race. 'It is enough', a shrewd man said, 'if a parable goes upon all fours; you must not expect it to draggle its tail on the ground as well.' Theories are made for use, to lighten the task of the mind in comprehending Nature, and difficulty is almost as great a fault as crudity; if crudity limits their use, difficulty, on the other hand, forbids it.

* As a Fellow of New College remarked to me, after the lecture.

It was in the problem of explaining a laboratory experiment that this high aspiring theory, which any poet might envy, took its rise. Aberration of light is consistent with an aether which drifts through the body of the earth undisturbed. The Michelson-Morley experiment is consistent with an aether which clings to the Earth's surface and moves with it. Naturally each is consistent with other suppositions, but where shall we find a supposition that is consistent with both? A paradox may require a paradoxical answer, and Einstein has supplied one. Put into common language it appears to me to say: Both phenomena belong to an indeterminate class, for they can be reproduced in calculation while employing a fictitious, local, peculiar time which does not represent all we know but is vitiated by a foreign addition. The experiments are oriented so that we have separated ourselves from our ordinary means of standardizing time. One suggests that the aether is sweeping through the earth's surface, and the other that it is still. But in rigour we can draw no conclusion from them. In just the same way, in geometry, if we set out to assign the position of the vertices of the triangle inscribed to one circle and circumscribed to another, which upon general grounds we should expect to be definite, we might arrive by experiment at apparently conflicting results. The explanation is that the data of the problem are peculiar, and contrary to anticipation, do not involve the complete number of facts required for a determinate result.

It must not be supposed that we are tied by necessity to experiments that prove to be inconclusive in principle. The finite velocity of light was discovered from observations of Jupiter's Satellites, by just one of those repetitive processes upon which I have laid stress. Roemer examined the period intervening between successive

eclipses of Satellite I at different points of the Earth's orbit. It then appeared that the average was less when the Earth was approaching Jupiter than when they were separating from one another. I have never heard of any explanation of this fact except that the Earth's velocity in its orbit anticipates the receipt of the light-pulse in the former case and defers it in the latter. The velocity inferred is, in fact, treated in a modern discussion as a method of determining the dimensions of the Earth's orbit. With modern observations of the moment of eclipse, extending over twenty-five years or so, and thus introducing plenty of repetitions in all the circumstances, the degree of certainty arrived at by this velocity method is rough, by astronomical standards, but far from negligible; the value of the solar parallax is in close agreement with that found by more purely geometrical methods—such as the position of the planet Eros,—with a 'probable error' of one part in four hundred. Ten times as great a precision would be wanted before we could definitely distinguish by these means a relative velocity of transmission $c$ from one of value $c \pm u$, where $c$ is the velocity of light, terrestrially determined, and $u$ is the velocity of the Earth in its orbit, but in principle I presume that we could do so. I suppose that the result as far as it goes would allow a transmission of light across space which falsified Newtonian axioms of relative velocity, as it certainly allows one that agrees with them. But if the Relativist desires the former scheme, I must leave to his ingenuity to describe the associated geometry and time system that he would employ, as I profess myself unable to imagine them. His device of 'local time' seems to me completely excluded by the repetitions.

There is a passage in Gibbon's *Autobiography* which

has always interested me; speaking of his mathematical studies, 'as soon as I understood the principle I relinquished for ever the pursuit of mathematics, nor can I lament that I desisted before my mind was hardened by the habit of rigid demonstration, so destructive of the finer feeling of moral evidence, which must however determine the actions and opinions of our lives'. There may be some malice in that; possibly Gibbon was thinking of some dear friend when he wrote it. But we cannot deny that it conveys a salutary reminder. Mathematics no less than other aids to thought lives in a world where actuality is bartered for convenience. We must recognize the risk we take if we follow it; it may carry us farther than we are entitled to go. As time extends knowledge, it extends equally the unknown. We must be content to read the new page, as we read the old, with our finger-tips, like a blind man, not knowing what comes next.

I have the privilege this evening of speaking under the protection of Halley's genial name. Halley was a learned mathematician and he was a great humanist, a man of strong practical sense. We owe to him the production of Newton's *Principia*. Our obligation to his common sense, to say the least, in getting it done, is probably greater than we shall ever know. In praising common sense to-day I would not seem to withhold any term of admiration for the genius which has for the first time proved capable of rehandling Newton's gravitation, and has given us the bold, new, subtle, soaring theory of relativity. But as for myself, I have thought right in addressing you to endeavour to show that we may if we please continue to keep our feet upon this earth.

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